Investment Subsidies and Redistributive Capital Income Taxation in a Neoclassical Growth Model

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Motivation

- Distortionary taxes and economic growth.
- Capital income taxes, investment, and pure redistribution.

Motivation

- The role of investment stimulation
  - on redistribution and on investment.
  - and its relation to
    - economic crises and
    - long-run effects.
- Distortionary taxes, redistribution, investment subsidies and (neoclassical) economic growth.

Relation to Literature

- The Judd (1985), Chamley (1986) “celebrated result“ for neoclassical growth:
  1. *Capital income taxes should be zero in the long run.*
  2. *Capital income taxes are bad instruments for (pure) redistribution.*

Relation to Literature

- Counterexamples and extensions
  - E.g. Lansing (1999), Guo and Lansing (1999)
    - using a Solow setup
  - E.g. Uhlig and Yanagawa (1996), Rehme (1995), (2002)
    - using endogenous growth setups.
  - E.g. Jones et al. (1997)

- What should be taxed?
  - E.g. Fisher (1937), Kaldor (1955)

The Model

- Agents
  - Identical competitive firms
  - Infinitely lived, price taking workers and capitalists
    - Classical savings rule.
    - Thus, two class model structure à la Kaldor (1957).
  - The government
    - taxes capital income.
    - grants investment subsidies and redistributes.
- No uncertainty, no technical progress, no population growth, no depreciation.
- Inelastic labour supply
  - Follows Judd (1985)

The capital owners

- The capital owners' instantaneous budget constraint

$$c_t + i_t = (1 - \theta_t) r_t k_t + p_t i_t \quad \text{and} \quad i_t = \dot{k}_t.$$

c_t	- consumption	i_t	- (net) investment
r_t	- rate of return	k_t	- capital
θ_t	- capital income tax	p_t	- investment subsidy

The capital owners

- The capital owners' problem

$$\max_{c_t^k} \int_0^\infty u[c_t]\, e^{-\rho t} dt$$

$$s.t. \qquad \dot{k}_t = \left(\frac{1-\theta_t}{1-p_t}\right) r_t k_t - \frac{c_t}{1-p_t} \qquad (1)$$

$$k(0) = \text{given}, \quad k(\infty) = \text{free}.$$

- $u[c_t]$ satisfies standard properties.

The capital owners

- CV Hamiltonian

$$H = u[c_t] + \lambda_t \left(\left(\frac{1-\theta_t}{1-p_t} \right) r_t k_t - \frac{c_t}{1-p_t} \right)$$

- The necessary FOCs

$$H_c: \quad u' - \tfrac{\lambda_t}{1-p_t} = 0 \tag{2a}$$

$$H_k: \quad -\lambda_t \left(\tfrac{1-\theta_t}{1-p_t} \right) r_t + \rho\lambda_t = \dot{\lambda}_t \tag{2b}$$

plus $\lim\limits_{t\to\infty} k_t \lambda_t e^{-\rho t} dt = 0$ and that (1) holds.

- λ_t : the capital owners' shadow price of an additional unit of capital in terms of utility.

The workers

- The workers do not invest, are not taxed by assumption, and supply labour inelastically.
- They consume their entire income x_t.
- x_t depends wage and lump-sum transfer income

$$x_t = w_t + TR_t. \tag{3}$$

- Intertemporal utility

$$\int_0^{\infty} v[x_t]\, e^{-\rho t} dt$$

where $v[x_t]$ satisfies standard properties.

The firms

- The firms are owned by capital owners, they face perfect competition and maximize profits.
- Aggregate technology is CRTS.
- Profit maximization implies

$$r_t = f'(k_t) \tag{4}$$

$$w_t = f(k_t) - f'(k_t)k_t \tag{5}$$

- Perfect competition implies zero profits.

The government

- The government chooses θ_t, p_t and TR_t under the balanced budget condition

$$TR_t = \theta_t r_t k_t - p_t \dot{k}_t.$$

  - TR_t - lump-sum transfers to workers
  - θ_t - tax rate on capital income
  - p_t - fraction of investment that is subsidized by the government.
- Hence, capital-cum-investment-subsidy-tax (CICIST) scheme.
- On capital income taxes and consumption taxes see e.g. Judd (1999).

The private sector: Arbitrary Behaviour

- Consider arbitrary, not necessarily optimal behaviour, but obeyance of budget constraints.
- Then, we have for
  - the workers

$$x_t = w_t + TR_t = f(k_t) + r_t k_t + \theta_t r_t k_t - p_t \dot{k}_t$$

  - capitalists

$$\dot{k}_t = \left(\frac{1-\theta_t}{1-p_t} \right) r_t k_t - \frac{c_t}{1-p_t}$$

The private sector: Arbitrary Behaviour

- Consider arbitrary, not necessarily optimal behaviour, but obeyance of budget constraints.
- Then, the model implies that i.a. and ceteris paribus
  1. $\frac{dx_t}{dp_t}_{|\theta_t, c_t} \leq 0$: Higher investment subsidies seem to be bad for redistribution and so the workers' income.
  2. $\frac{dx_t}{d\theta_t}_{|p_t, c_t} > 0$: Higher capital income taxes are good for redistribution and so the workers' income.
  3. $\frac{d\dot{k}_t}{d\theta_t}_{|p_t, c_t} < 0$: Higher capital income taxes are bad for investment.
  4. $\frac{d\dot{k}_t}{dp_t}_{|\theta_t, c_t} \geq 0$: Higher investment subsidies seem to be good for investment.

Investment Return Stabilization

- The return to real investment

$$R_t \equiv \frac{(1-\theta_t)r_t}{(1-p_t)}. \tag{6}$$

- Suppose due to a crisis there is a sharp drop in the real return r_t.
- The government reacts by changing p_t or θ_t to keep R_t constant.

$$dR_t = 0 = R_r\ dr_t + R_p\ dp_t + R_\theta\ d\theta_t.$$

- Keeping R_t and the other policy instrument constant

$$\frac{dp_t}{dr_t} = -\frac{(1-p_t)}{r_t} \tag{7}$$

$$\frac{d\theta_t}{dr_t} = \frac{(1-\theta_t)}{r_t} \tag{8}$$

Investment Return Stabilization

- Thus, for the (arbitrary) policy objective to stabilize the real investment return:

Proposition When there is a *drop* in the real return to capital r_t, and if the government wishes to 'stabilize' the real return to investment, the government should *increase* the investment subsidy p_t or *cut* the capital income tax rate θ_t by compensating amounts, respectively.

Investment Return Stabilization. Example

- Before crisis $r_t = 0.05$, and $\theta_t = 0.35$ as in U.S.
- Consider a conservative $p_t = 0.25$.
- Suppose r_t drops by 50 percent.

$$\frac{dp_t}{p_t} = \left(\frac{-(1-p_t)}{p_t}\right) \cdot \frac{dr_t}{r_t} = (-0.75/0.25) \cdot (-0.50) = 1.50.$$

  Hence, p_t should be more than doubled, i.e. be raised to 0.625.

- $$\frac{d\theta_t}{\theta_t} = \left(\frac{(1-\theta_t)}{\theta_t}\right) \cdot \frac{dr_t}{r_t} = (0.65/0.35) \cdot (-0.50) = -0.93.$$

  Thus, the tax rate should be reduced to almost zero.

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- Difference: Political implementation procedures!

Non-distortion of accumulation

- Recall the capital owners' FOC (30b)

$$-\lambda_t \left(\frac{1-\theta_t}{1-p_t} \right) r_t + \rho \lambda_t = \dot{\lambda}_t$$

- The distortion of accumulation in long-run equilibrium $\dot{\lambda}_t = 0$ depends on

$$\left(\frac{1-\theta_t}{1-p_t} \right)$$

- Non-distortion if
  1. $\theta_t = 0$ and $p_t = 0$. (Judd (1985), Chamley (1986))
  2. $\theta_t = p_t$. (This paper)

Non-distortion of accumulation

- If $\theta_t = p_t$, then

$$x = w + TR = f(k) - rk + \theta rk - \theta \dot{k} \tag{9}$$

- Substitution of (1) into (9) one then obtains

$$x = f(k) - rk + \frac{\theta c}{1-\theta} \tag{10}$$

- Equilibrium income of the workers is increasing in the consumption of the capital owners and in θ.

The government's problem

- The government respects the private sector optimality conditions,
  - keeps the agents on their respective supply and demand curves,
  - chooses a policy that can be realized as a competitive equilibrium.

The government's problem

$$\max_{k,c,\theta,p,\lambda} \int_0^\infty \left\{ \gamma\, v \left[f(k) - \left(\frac{1-\theta}{1-p} \right) rk + \frac{pc}{1-p} \right] + u[c] \right\} e^{-\rho t} dt$$

s.t.

$$u'(c) - \frac{\lambda}{1-p} = 0 \tag{11a}$$

$$-\left(\frac{1-\theta}{1-p}\right) r\lambda + \rho\lambda = \dot{\lambda} \tag{11b}$$

$$\left(\frac{1-\theta}{1-p}\right) rk - \frac{c}{1-p} = \dot{k} \tag{11c}$$

$$\theta, p_t \geq 0 \;\text{ and }\; \lim_{t\to\infty} \lambda k e^{-\rho t} = 0 \tag{11d}$$

- $\gamma \in (0, \infty)$: social weight attached to the welfare of the workers.
  - If $\gamma \to 0(\infty)$, the government is only concerned about the capitalists (workers).

The government's problem

- Current value Hamiltonian

$$
\begin{aligned}
\mathcal{H} &= \gamma v[\cdot] + u[c] \\
&\quad + \mu_1 (u' - \frac{\lambda}{1-p}) + q_1 \lambda \left(- \left(\frac{1-\theta}{1-p} \right) r + \rho \right) \\
&\quad + q_2 \left(\left(\frac{1-\theta}{1-p} \right) r\, k - \frac{c}{1-p} \right)
\end{aligned}
$$

  - q_1: *social* marginal value of the *private* marginal value λ
  - λ: how valuable is more capital is in terms of utility.
  - q_2: *social* marginal value of more capital k.

The government's problem

- The necessary FOCs

$$\mathcal{H}_k: \quad \gamma\, v'[\cdot]\left(f' - \left(\frac{1-\theta}{1-p}\right) r\right) + q_2\left(\frac{1-\theta}{1-p}\right) r = \rho q_2 - \dot{q}_2 \qquad (12a)$$

$$\mathcal{H}_c: \quad \gamma\, v'[\cdot]\frac{p}{1-p} + u'[\cdot] + \mu_1 u''[\cdot] - q_2\frac{1}{1-p} = 0 \qquad (12b)$$

$$\mathcal{H}_\theta: \quad \theta\left\{\gamma v'[\cdot]\frac{rk}{(1-p)} + q_1\lambda\frac{r}{1-p} - q_2\frac{rk}{(1-p)}\right\} = 0 \qquad (12c)$$

$$\mathcal{H}_p: \quad p\left\{(\gamma v'[\cdot] - q_2)\left[\frac{c-(1-\theta)rk}{(1-p)^2}\right] - \lambda\left(\frac{\mu_1 + q_1 r(1-\theta)}{(1-p)^2}\right)\right\} = 0 \qquad (12d)$$

$$\mathcal{H}_\lambda: \quad -\frac{\mu_1}{1-p} + q_1\left(-\left(\frac{1-\theta}{1-p}\right) r + \rho\right) = \rho q_1 - \dot{q}_1 \qquad (12e)$$

- $+$ transversality conditions $+$ constraints.

The government's problem

- Focus on interior solutions.
- At time zero, the initial λ is unconstrained.
- Thus, the associated costate variable q_1 at time 0 is zero, i.e. $q_1(0) = 0$.
- Rearrangement implies

$$
\begin{aligned}
\mathcal{H}_\theta : (\gamma v' - q_2)\frac{rk}{1-p} &= -q_1 \, \lambda \, \frac{r}{1-p} \\
(\gamma v' - q_2) &= -q_1 \, \frac{\lambda}{k} \qquad (13) \\
\mathcal{H}_p : (\gamma v' - q_2)\frac{c-(1-\theta)rk}{(1-p)^2} &= \lambda \, \frac{\mu_1 + (1-\theta)r \, q_1}{(1-p)^2} \qquad (14)
\end{aligned}
$$

The government's problem

- Substitute for $(\gamma v' - q_2)$ from (13) in (14) to get

$$\begin{aligned}\mathcal{H}_p : -q_1 \frac{\lambda}{k}\left(\frac{c-(1-\theta)rk}{(1-p)^2}\right) &= \lambda \frac{\mu_1 + (1-\theta)rq_1}{(1-p)^2} \\ -q_1 \frac{c}{k} &= \mu_1 . \qquad (15)\end{aligned}$$

- Substitute this in (12e) to get

$$q_1 \frac{c/k}{1-p} + q_1\left(-\left(\frac{1-\theta}{1-p}\right) r + \rho\right) = \rho q_1 - \dot{q}_1 .$$

- This is a homogeneous, linear differential equation.

The government's problem

- Solve to get

$$\begin{aligned} q_1(t) &= q_1(0)e^{-\int_0^t \Delta_s ds} \\ \text{where} \quad & \Delta_s \equiv \left[\frac{c/k}{1-p} - \left(\frac{1-\theta}{1-p}\right) r\right] \\ \text{and} \quad & q_1(0) = 0 \end{aligned} \tag{16}$$

- Hence,

  Lemma 1 $q_1(t) = 0$ for all $t \in [0, \infty)$.

The government's problem

- The necessary FOCs

$$\begin{aligned}
\mathcal{H}_k: &\quad \gamma\, v'[\cdot]\left(f' - \left(\tfrac{1-\theta}{1-p}\right) r\right) + q_2\left(\tfrac{1-\theta}{1-p}\right) r = \rho q_2 - \dot{q}_2 \\
\mathcal{H}_c: &\quad \gamma\, v'[\cdot]\tfrac{p}{1-p} + u'[\cdot] + \mu_1 u''[\cdot] - q_2\tfrac{1}{1-p} = 0 \\
\mathcal{H}_\theta: &\quad \theta\left\{\gamma v'[\cdot]\tfrac{rk}{(1-p)} + q_1\lambda\tfrac{r}{1-p} - q_2\tfrac{rk}{(1-p)}\right\} = 0 \\
\mathcal{H}_p: &\quad p\left\{(\gamma v'[\cdot] - q_2)\left[\tfrac{c-(1-\theta)rk}{(1-p)^2}\right] - \lambda\left(\tfrac{\mu_1+q_1 r(1-\theta)}{(1-p)^2}\right)\right\} = 0 \\
\mathcal{H}_\lambda: &\quad -\tfrac{\mu_1}{1-p} + q_1\left(-\left(\tfrac{1-\theta}{1-p}\right) r + \rho\right) = \rho q_1 - \dot{q}_1
\end{aligned}$$

- $+$ transversality conditions $+$ constraints.

The government's problem

- Now we look at the long run.
- Long-run equilibrium if

$$\dot{k} = \dot{\lambda} = \dot{c} = \dot{q}_1 = \dot{q}_2 = 0$$

- From (11b) with $\dot{\lambda} = 0$ we have

$$\lambda \left(\rho - r \left(\frac{1-\theta}{1-p} \right) \right) = 0 \quad \textit{where} \quad \lambda \geq 0. \tag{18}$$

  Substituting in (12a) implies

$$\gamma v'[\cdot] \left(f' - \frac{1-\theta}{1-p} r \right) = 0$$

  must hold and profit maximization implies $f' = r$.
- But then we must have $\theta = p$ in an optimum.

The government's problem

Proposition 1 No matter whether the government is relatively more pro-labour or pro-capital, the optimal policy under the capital-income-cum-investment-subsidy-tax (CICIST) scheme is not to distort capital accumulation by setting $\theta = p$.

Non-distortion of accumulation and the optimum

- When $\theta_t = p_t$, then

$$x = w + TR = f(k) - rk + \theta rk - \theta \dot{k}$$

- Substitution of (1) into (9) one then obtains

$$x = f(k) - rk + \frac{\theta c}{1 - \theta}$$

- Equilibrium income of the workers is increasing in the consumption of the capital owners and in θ.

The government's problem

- Next, the FOCs imply that θ must solve

$$\gamma\, v'[f(\tilde{k}) - \rho\tilde{k} + \theta\rho\tilde{k}] = u'[(1-\theta)\rho\tilde{k}]. \tag{19}$$

  where $\tilde{k}$ is the steady state capital stock.
- As $\gamma \to \infty$, $\theta = 1$ is optimal, since $\lim_{c_t \to 0} u'[\cdot] = \infty$.
- If $\gamma \to 0$, then $\theta = 0$ is optimal. See eq. (12c)

The government's problem

Lemma 2 If the workers and the capitalists have different utility functions under the CICIST scheme and

1. the government represents the capitalists only ($\gamma \to 0$), then the optimal capital income tax under CICIST is zero in the long run and redistribution from capital to labour is zero.
2. the government represents the workers only ($\gamma \to \infty$), then the optimal capital income tax under CICIST is nonzero in the long run and redistribution from capital to labour is maximal.

The government's problem

- Now assume CRRA utility

$$u[c] = \frac{c^{1-\beta} - 1}{1-\beta} \quad \textit{and} \quad v[x] = \frac{x^{1-\beta} - 1}{1-\beta}.$$

- Then by (19) θ has to solve

$$\begin{aligned} \gamma \left(f(\tilde{k}) - (1-\theta)\rho\tilde{k} \right)^{-\beta} &= \left((1-\theta)\rho\tilde{k} \right)^{-\beta} \\ \frac{f(\tilde{k})}{(1-\theta)\rho\tilde{k}} &= \gamma^{\frac{1}{\beta}} + 1. \end{aligned}$$

The government's problem

- As $r = \rho = f'$, we have $\frac{\rho\tilde{k}}{f(\tilde{k})} \equiv \alpha$.
- Thus, θ has to solve

$$\tilde{\theta} = \frac{\alpha(\gamma^{\frac{1}{\beta}} + 1) - 1}{\alpha(\gamma^{\frac{1}{\beta}} + 1)} \tag{20}$$

- $\tilde{\theta}$ is increasing in the capital share α.
- Thus, distribution matters.

The government's problem

- $\tilde{\theta} > 0$ if

$$\gamma > \left(\frac{1-\alpha}{\alpha} \right)^{\beta}. \tag{21}$$

- Thus, a positive $\tilde{\theta}$ depends on γ, α and β.

The government's problem

Proposition 2 Let the agents possess the same constant relative risk aversion utility functions. Under a capital-income-cum-investment-subsidy-tax (CICIST) scheme the optimal capital income tax rate $\tilde{\theta}$ is non-zero if the social planner attaches sufficient weight on the welfare of the workers $\gamma > \left(\frac{1-\alpha}{\alpha}\right)^{\beta}$. In contrast, if $\gamma < \left(\frac{1-\alpha}{\alpha}\right)^{\beta}$, then $\tilde{\theta} = 0$ is optimal. Hence, under CICIST the income distribution, preferences and the political weight of the workers determine whether the optimal capital income taxes are zero in the long run.

Simulation exercise

Table: Baseline Parameter Values

α	ρ	β
0.36	0.011	2

Based on Walsh (2003), p. 75, for quarterly U.S. data

- If $\gamma > \left(\frac{1-\alpha}{\alpha}\right)^{\beta} = 3.2$, then $\tilde{\theta} > 0$.

Simulation exercise

- The optimal capital income tax rate as a function of γ.

Table: Optimal Capital Income Tax Rates $\widetilde{\theta}$

γ	5	10	15	20	50	80
$\widetilde{\theta}$	0.14	0.33	0.43	0.49	0.66	0.72

γ	100	200	500	1000	10000
$\widetilde{\theta}$	0.75	0.82	0.88	0.91	0.97

Conclusion

- Coupling capital income taxes with investment subsidies in a neoclassical growth environment may imply positive capital income tax rates in the long-run optimum.
- This holds for a large class of utility functions.
- Capital income taxes may not be bad instruments for pure redistribution.
- The conditions for optimal, long-run positive tax rates are quite realistic.
  1. Political power of transfer receivers.
  2. Inequality in pre-tax factor incomes.
  3. Preferences: Intertemporal elasticity of substitution.

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EXAMPLE:

A model with Accelerated Depreciation Allowances

The capital owners

- The capital owners' instantaneous budget constraint

$$c_t + i_t = r_t k_t - T_t \quad \text{and} \quad i_t = \dot{k}_t + \delta k_t, \tag{22}$$

c_t	- consumption	i_t	- (gross) investment
r_t	- rate of return	k_t	- capital
T_t	- taxes paid by capital owners	δ	- 'true' capital depreciation

- The capital owners' intertemporal utility

$$\int_0^\infty u[c_t]\, e^{-\rho t} dt \tag{23}$$

where $u[c_t]$ satisfies the usual properties.

The workers

- The workers do not invest and are not taxed by assumption.
- Inelastic labour supply.
- They consume their entire income.
- Their total income x_t depends on wages, w_t, and lump-sum transfer income, TR_t,

$$x_t = w_t + TR_t. \tag{24}$$

- The workers' intertemporal utility

$$\int_0^{\infty} v[x_t] \, e^{-\rho t} dt$$

where $v[x_t]$ satisfies standard properties.

The firms

- The firms face perfect competition and maximize profits.
- Aggregate technology is CRTS.
- Profit maximization implies

$$r_t = f'(k_t) \tag{25}$$

$$w_t = f(k_t) - f'(k_t)k_t \tag{26}$$

- Perfect competition implies zero profits.

The government

- The government taxes capital income net of a depreciation allowance.
- Consider accelerated depreciation of capital, D_t, as in Sinn (1987).

$$D_t \equiv p_t i_t + (1 - p_t)\delta k_t = p_t \dot{k}_t + \delta k_t. \tag{27}$$

where $0 \leq p_t \leq 1$ of an investment is depreciated immediately and $(1 - p_t)$ gradually over time.
- The government taxes capital income net of the depreciation allowance and, from the resulting tax revenues, grants (unproductive) transfers to the workers.
- The government budget constraint

$$T_t = \theta_t \left[r_t k_t - p_t \dot{k}_t - \delta k_t \right] = TR_t \tag{28}$$

where θ_t is the tax rate on (net) capital income.

The private sector: Capital owners

- The capital owners' problem

$$\max_{c_t^k} \int_0^\infty u[c_t] \; e^{-\rho t} dt$$

$$s.t. \quad \dot{k}_t = \frac{(1-\theta_t)(r_t-\delta)k_t - c_t}{(1-\theta_t p_t)} \quad \text{and } k(0) = \text{given}, \qquad (29)$$

- FOCs

$$H_c : \qquad u' - \frac{\lambda_t}{1-\theta_t p_t} = 0 \qquad (30a)$$

$$H_k : \quad -\lambda_t \left(\frac{(1-\theta_t)(r_t-\delta)}{1-\theta_t p_t} \right) + \rho\lambda_t = \dot{\lambda}_t \qquad (30b)$$

plus $\lim_{t\to\infty} k_t \lambda_t e^{-\rho t} = 0$ and that equation (29) holds.

- λ: the capital owners' shadow price of an additional unit of capital in terms of utility.

The private sector: Workers

- The workers' income

$$x_t = w_t + TR_t = f(k_t) - r_t k_t + \theta_t \left[r_t k_t - p_t \dot{k}_t - \delta k_t \right] \qquad (31)$$

- Substitution and simplification yield

$$x_t = f(k_t) - \left(\frac{1-\theta_t}{1-\theta_t p_t} \right) r_t k_t - \frac{\theta_t (1-p_t) \delta k_t}{1-\theta_t p_t} + \frac{\theta_t p_t c_t}{1-\theta_t p_t}. \qquad (32)$$

- x_t is increasing in c_t.

The private sector: Arbitrary Behaviour

- Consider arbitrary, not necessarily optimal behaviour, but obeyance of budget constraints.
- Then, the model implies that i.a. and ceteris paribus
  1. $\frac{dx_t}{dp_t}_{|\theta_t, c_t} \leq 0$: Higher depreciation allowances seem to be bad for redistribution and so the workers' income.
  2. $\frac{dx_t}{d\theta_t}_{|p_t, c_t} > 0$: Higher capital income taxes are good for redistribution and so the workers' income.
  3. $\frac{d\dot{k}_t}{d\theta_t}_{|p_t, c_t} < 0$: Higher capital income taxes are bad for investment.
  4. $\frac{d\dot{k}_t}{dp_t}_{|\theta_t, c_t} \geq 0$: Higher depreciation allowances are good for investment.

Investment Return Stabilization

- The return to real investment

$$R_t \equiv \frac{(1-\theta_t)(r_t-\delta)}{(1-\theta_t p_t)}. \tag{33}$$

- Suppose due to a crisis there is a sharp drop in the real return r_t.
- The government reacts by changing p_t or θ_t to keep R_t constant.

$$dR_t = 0 = R_r\, dr_t + R_p\, dp_t + R_\theta\, d\theta_t.$$

- Keeping R_t and the other policy instrument constant

$$\frac{dp_t}{dr_t} = -\frac{(1-\theta_t p_t)}{\theta_t(r_t-\delta)} \tag{34}$$

$$\frac{d\theta_t}{dr_t} = \frac{(1-\theta_t)(1-\theta_t p_t)}{(r_t-\delta)(1-p_t)} \tag{35}$$

Investment Return Stabilization

- Thus, for the (arbitrary) policy objective to stabilize the real investment return:

Proposition When there is a *drop* in the real return to capital r_t, the government should *increase* the accelerated capital depreciation allowance p_t or *cut* the capital income tax rate θ_t by compensating amounts, respectively, if it wishes to ‘stabilize’ the real return to investment.

Non-distortion of accumulation

- The capital owners' accumulation decision is governed by the Euler equation

$$-\lambda_t \left(\frac{(1-\theta_t)(r_t-\delta)}{1-\theta_t p_t} \right) + \rho\lambda_t = \dot{\lambda}_t.$$

- Distortion is due to

$$\frac{1-\theta_t}{1-\theta_t p_t}.$$

- Non-distortion if

  - $\theta_t = 0$ and/or $p_t = 1, \forall t$.

  1. $\theta(\infty) = 0$ and $p_t = 0$. E.g. Judd (1985), Chamley (1986)
  2. $\theta(\infty) \geq 0, p(\infty) = 1$. See Rehme (2011).